

Part I: Dimensional Analysis (aka Factor-Label or Unit Cancellation Method)

Sample Problem 1

A large, coal-fired electric power plant produces 12 million kilowatt-hours of electricity each day. Assume that an input of 10,000 BTUs of heat is required to produce an output of 1 kWh of electricity.

- Calculate** the number of BTUs of heat needed to generate the electricity produced by the power plant each day.
- Calculate** the number of pounds of coal consumed by the power plant each day, assuming that one pound of coal yields 5,000 BTUs of heat.
- Calculate** the number of pounds of sulfur released by the power plant each day, assuming that the coal contains one percent sulfur by weight.

Where to start?

Step 1: Ask yourself, “What unit of measure do I need for my answer?”

Read part (a) again. You need to answer in units of “BTUs *per* day”. The word “*per*” is code for “divided by” or “in the denominator”. Next, translate the words into math. So, “BTUs per day” becomes $\frac{\text{BTU}}{\text{day}}$.

Consider this your *end goal*; it is how you must express your final answer.

Step 2: Am I given any numbers that are paired with the units I seek?

Not exactly! That would be *WAY* too easy. But you can identify a “bridge” of sorts that connects “BTU” and “day”. You are given “power plant produces 12 million kilowatt-hours (12,000,000 kWh) of electricity each day” AND “10,000 BTU” of heat is required to produce an output of 1 kWh of electricity”. From this we can write two equivalent sets of conversion factors for each of these tidbits of valuable information.

$$\frac{12,000,000 \text{ kWh}}{1 \text{ day}} \text{ or } \frac{1 \text{ day}}{12,000,000 \text{ kWh}} \text{ as well as } \frac{1 \text{ kWh}}{10,000 \text{ BTU}} \text{ or } \frac{10,000 \text{ BTU}}{1 \text{ kWh}}$$

These are called *conversion factors* and they are your friends! Prepositions are also your friends since they link information together as in “pieces of candy” and “at 15 °F”. Your objective is to string the conversion factors together (keeping the numbers with the units) so that the *unwanted units* cancel out leaving you with the desired units for your answer. In our example the desired units are $\frac{\text{BTU}}{\text{day}}$. The numerator and denominator of a

conversion factor represent the same quantity, such as $\frac{12 \text{ inches}}{1 \text{ foot}}$ or $\frac{1 \text{ foot}}{12 \text{ inches}}$ thus we are simply multiplying by “1”.

Step 3: Choose the conversion factors AND structure them so that you achieve your “end goal”.

Remember we were asked to solve for “BTUs per day” which in “math speak” looks like units of $\frac{\text{BTU}}{\text{day}}$.

It there is a “trick” to this process, it involves *choosing* conversion factors and arranging them so that the *unwanted* units cancel out leaving only the *desired* unit(s) on your final answer! For our example, you want BTU in the numerator of your final answer along with days in the denominator. So, you need to get rid of or cancel the kWh. How?

$\frac{12,000,000 \cancel{\text{kWh}}}{1 \text{ day}}$ and $\frac{10,000 \text{ BTU}}{1 \cancel{\text{kWh}}}$ make a nice pair since they result in the desired units of $\frac{\text{BTU}}{\text{day}}$. Yippee!

Step 4: Solve it! Let’s revisit our sample problem:

When you have cancelled out the units you don't want and are left only with the units you do want, then you know it's time to multiply all the numerators (top numbers) together, and divide by EACH of the bottom numbers! When setting up a string of conversion factors, it is often helpful to start at the end (units of the desired answer) and work backwards as explained and demonstrated on the screencast.

Sample Problem 1

A large, coal-fired electric power plant produces 12 million kilowatt-hours of electricity each day. Assume that an input of 10,000 BTUs of heat is required to produce an output of 1 kWh of electricity.

- (a) **Calculate** the number of BTUs of heat needed to generate the electricity produced by the power plant each day.

- (b) **Calculate** the number of pounds of coal consumed by the power plant each day, assuming that one pound of coal yields 5,000 BTUs of heat. (Pick up where you left off! Let the problem lead you!)

- (c) **Calculate** the number of pounds of sulfur released by the power plant each day, assuming that the coal contains one percent sulfur by weight.

Sample Problem 2

The environmental impact of washing a load of dirty dishes in an electric dishwasher differs from that of washing them in a sink by hand. A comparison of the two methods may allow consumers to spend their money more wisely. Use the assumptions in the table below to perform the calculations that follow.

All the dishes fit into a single load.
The water entering both the water heater and the sink is at 50 °F.
The water heater and the dishwasher are both 100% efficient.
Washing the dishes by hand requires 20 gallons of water heated to 110 °F.
In one complete cycle, the electric dishwasher uses 10 gallons of water heated to 140 °F and the dishwasher also uses 0.500 kilowatt-hour of electrical energy for its mechanical operations.

- (a) A British thermal unit or BTU is the amount of energy needed to raise the temperature of one pound of water 1.0 °F. Additionally, one gallon of water has a mass of 8.0 pounds. **Calculate** the total energy used to wash a load of dishes using the dishwasher.
- (b) One kilowatt-hour is equivalent to 3,400 BTUs and the cost of electricity is \$0.11 per kWh. **Calculate** the energy in BTU used to heat the water for washing a load of dishes by hand.
- (c) **Calculate** the cost of electricity for each dishwashing method given that one kilowatt-hour is equivalent to 3,400 BTUs and the cost of electricity is \$0.11 per kWh.

Step 1: Ask yourself, “What unit of measure do I need for my answer?”

Read part (a) again. The question asks you to calculate “total energy” per load, so you need to answer in units of “BTU per load”. Translate that to math and you get $\frac{\text{BTU}}{\text{load}}$ or simply BTU. Consider this your *end goal*; it is how you must express your final answer for parts (a). Next, read part (b) again. The final answer to part (b) should also be reported in $\frac{\text{BTU}}{\text{load}}$ or simply BTU.

Step 2: Am I given any numbers that are paired with the units I seek?

Yes! (Remember, it is a legal maneuver to “flip” any of these conversion factors as long as you keep the appropriate number with the appropriate unit!)

$$\frac{10 \text{ gal H}_2\text{O}}{\text{load}} \text{ and } \frac{8 \text{ gal H}_2\text{O}}{1 \text{ lb H}_2\text{O}} \text{ and } \frac{1 \text{ BTU}}{1 \text{ lb H}_2\text{O} \cdot 1 \text{ }^\circ\text{F}} \text{ and note the temperature change!}$$

Yep! That’s TWO items in the denominator of one of the conversion factors, thus twice the cancelling fun.

Step 3: Choose the conversion factors AND structure them so that you achieve your “end goal”.

Ask yourself “What to flip and why flip at all?”

Step 4: Solve it!

Sample Problem 2

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Part 2: Determining Energy Efficiency

Energy efficiency is simply the ratio of work output to work input. *If* the problem states that system is less than 100 % efficient, then the energy output must be *divided* by the stated efficiency to determine how much energy input is needed.

$$\text{Percent Efficiency} = \frac{\text{Work}_{\text{out}}}{\text{Work}_{\text{in}}} \times 100\%$$

Our previous sample problem made this easy on us by telling us the efficiency is 100%. Don't always expect that since such machines do not exist! If a machine or process is 75% efficient then you simply multiply anything you calculate as being "perfect" by 0.75 effectively reducing it to only 75% rather than leaving it at the elusive 100% efficiency.

Sample Problem 3

Answer the questions below regarding the heating of a house in the Midwestern United States. Use the assumptions in the table below to perform the calculations that follow.

The house has 2,000 square feet of living space.
80,000 BTUs of heat per square foot are required to heat the house for the winter.
Natural gas is available at a cost of \$5.00 per thousand cubic feet.
One cubic foot of natural gas supplies 1,000 BTUs of heat energy.
The furnace in the house is 80 percent efficient.

- (a) **Calculate** number of cubic feet of natural gas required to heat the house for one winter. Show all the steps of your calculations, including units.
- (b) **Calculate** the cost of heating the house for one winter.

Part 3: Scientific Notation

Scientific notation is used to communicate extremely large numbers such as the speed of light or extremely small numbers such as the radius of a blood cell.

1. Converting decimal numbers LESS THAN ONE to Scientific Notation

Example: Express the number 0.00234 in scientific notation.

- Place a decimal after the first nonzero number. We refer to this as the *coefficient*. *Coefficients* need to be between 1 and 9. The *coefficient* is then multiplied by ten raised to an *exponent*, 10^{exponent}
- Determine the *exponent* on the “10” by counting the number of places you move the decimal point. If you move the decimal to the right, the *exponent* will be negative. If you move the decimal point to the left, the *exponent* will be positive.

Negative exponents tell us how many times the number has been **divided by 10**

Positive exponents tell us how many times the number has been **multiplied by 10**

So, to express the number 0.00234 in scientific notation, we move the decimal 3 places to the right: 0.00234 to give 2.34×10^{-3}

What does this *really* mean mathematically? $\frac{2.34}{10 \times 10 \times 10}$ or $\frac{2.34}{10^3}$ or $\frac{2.34}{1,000}$ or 2.34×10^{-3}

Essentially, it means the number is small. How small? In the thousandths.

Why do this? For years you’ve thought 0.00234 was a perfectly good number just the way it is. Why convert it to scientific notation? BECAUSE it allows us to focus on the “order of magnitude” of *measurements* or *quantities* rather than the coefficients.

Compare 2.34×10^{-3} to 2.34×10^3

It’s not that the 2.34 is the most important part of the number—it’s that the number is in the **thousandths** (10^{-3}) versus the **thousands** (10^3). So, 2.34×10^3 is SIX power of ten’s ($10 \times 10 \times 10 \times 10 \times 10 \times 10$), that’s 10^6 , or a million times larger! We say that 2.34×10^3 is six orders of magnitude greater than 2.34×10^{-3}

2. Nuances of Adding and Subtracting Numbers Written in Scientific Notation

First, make sure both numbers have the same exponent on the “ $\times 10$ ” part. It’s easiest to convert one number to the greater exponent:

$$5.2 \times 10^3 + 3.6 \times 10^4 \quad \text{becomes} \quad 0.52 \times 10^4 + 3.6 \times 10^4 \quad \text{or, you may prefer it stacked} \quad \begin{array}{r} 0.52 \times 10^4 \\ + 3.6 \times 10^4 \\ \hline 4.12 \times 10^4 \end{array}$$

Either way, $(0.52 + 3.6) \times 10^4$ becomes 4.12×10^4

All done! Subtraction works the same way.

3. Nuances of Multiplying Numbers Written in Scientific Notation:

This is much easier! Simply, *multiply the coefficients* and then *add the exponents* on the power of 10.

$$(4 \times 10^{-2}) \times (2 \times 10^{10}) = (4 \times 2) \times 10^{(-2+10)} \text{ becomes } 8 \times 10^{(-2+10)} \text{ which simplifies to } 8 \times 10^8$$

Division works the same way EXCEPT you *subtract the exponents* on the power of 10.

$$(4 \times 10^{-2}) \div (2 \times 10^{10}) = (4 \div 2) \times 10^{(-2-10)} \text{ becomes } 2 \times 10^{(-2-10)} \text{ which simplifies to } 2 \times 10^{-12}$$

Sample Problem 4

Calculate the potential reduction in petroleum consumption in gallons of gasoline per year that could be achieved in the United States by introducing electric vehicles under the following assumptions:

- The mileage rate for the average car is 25 miles per gallon of gasoline.
- The average car is driven 10,000 miles per year.
- The United States has 150 million cars.
- Ten percent of the US cars could be replaced with electric cars.

Start with the easy simplification & translate the numbers into scientific notation:

10% of 150 million cars = 15 million cars = 1.5×10^7 cars... now on to the dimensional analysis...

Part 4: Population Growth Rate

Demography is the study of the characteristics of human populations, such as size, growth, density, distribution, and other vital statistics. In demography, population growth is used informally for the more specific term *population growth rate (PGR)*, and is often used to refer specifically to the growth of the human population of the world.

The most common way to express population growth is as a percentage, *not* as a rate. First, we calculate the *change* in a population using the difference between the crude birth and death rates. What are those?

The *crude birth rate (CBR)* is the total number of births per year per 1,000 people and the *crude death rate (CDR)* is the total number of deaths per year per 1,000 people. In the formula below we simply divide by ten to report the *PGR* as a percentage. (We wouldn't need to divide by 10 if the CBR & CDR were expressed per "100" instead of per 1,000.)

$$PGR \text{ (as a percentage)} = \frac{CBR - CDR}{10}$$

Part 5: The Rule of 70

The Rule of 70 is used to calculate the *time* required for a *doubling of a population* based upon population growth rate, *PGR*, expressed as a percentage. *Resist the urge to express the growth percentage as a decimal value, leave it as a percentage!*

The Rule of 70 is mathematically expressed as:

$$time_{doubling} = \frac{70}{PGR \text{ (expressed as a percentage)}}$$

How long will it take for a city with an annual population growth rate of 5% to double its population?

$$time_{doubling} = \frac{70}{PGR \text{ (expressed as a percentage)}} = \frac{70}{5\%} = 14 \text{ years}$$

How long will it take for the same city if the growth rate were 7%?

$$time_{doubling} = \frac{70}{PGR \text{ (expressed as a percentage)}} = \frac{70}{7\%} = 10 \text{ years}$$

Sample Problem 5

The fictional country **Industria** is tracking its population data. In 1855, the first year vital statistics were reported for the country, the population was 1.6 million, with a crude birth rate of 43 per 1,000. At that time the population of Industria was growing quite slowly, because of the high death rate of 41 per 1,000. In 1875 the population began to grow very rapidly as the birth rate remained at the 1855 level. While the crude death rate dropped dramatically to 20 per 1,000. Population growth continued to increase in the small country into the late 1800's, even though birth rates began to decline slowly.

In 1895 the crude birth rate had dropped to 37, and the crude death rate to 12 per 1,000. A complete census was also conducted and revealed that the population of Industria had grown to 2.5 million. By 1950 population growth gradually began to decline as the death rate remained at its 1895 level, while the birth rate continued to decline to 22 per 1,000. In 1977 vital statistics revealed that the death rate was 10 per 1,000, and that population growth had slowed even more to an annual rate of 0.4%. By 1990 Industria had reduced its birth rate to that of its now constant, low death rate, and the population transition was complete.

- Calculate** the annual growth rate of Industria in 1950.
- Calculate** the birth rate in Industria in 1977.
- Assume the population of Industria continues to maintain the population growth rate recorded for the country in 1895. **Determine** the year in which the expected size of the population will have quadrupled.

Part 6: Per Capita

Per Capita is a Latin term that translates into "by head," basically meaning "average per person." Per capita can even the place of saying "per person" in any number of statistical observances. In most cases the term is used in relation to economic data or reporting, but can also be used in most any other occurrence of population description.

Sample Problem 6

Between 1950 and 2000, global meat production increased from 52 billion kilograms to 240 billion kilograms. During this period, the global human population increased from 2.6 billion to 6.0 billion.

- (a) **Calculate** the per capita meat production in 1950 and in 2000.
- (b) **Calculate** the change in global per capita meat production during this 50-year period. Express your answer as a percentage.

Now let's see if you can put this newly acquired set of problem-solving skills to work!

Sample Problem 7

West Fremont is a community consisting of 3,000 homes. A small coal-burning power plant currently supplies electricity for the town. The capacity of the power plant is 12 megawatts (MW) and the average household consumes 8,000 kilowatt hours (kWh) of electrical energy each year. The price paid to the electric utility by West Fremont residents for the energy is \$0.10 per kWh. The town leaders are considering a plan, the West Fremont Wind Project (WFWP), to generate their own electricity using 10 wind turbines that would be located on the wooded ridges surrounding the town. Each wind turbine would have a capacity of 1.2 MW and each would cost the town \$3 million to purchase, finance, and operate for 25 years.

- (a) Assuming that the existing power plant can operate at full capacity for 8,000 hrs/yr, how many kWh of electricity can be produced by the plant in a year?
- (b) At the current rate of electricity energy per household, how many kWh of energy does the community consume in one year?
- (c) Assuming that the electrical energy needs of the community do not change during the 25-year lifetime of the wind turbines, what would be the cost to the community of the electricity supplied by the WFWP over 25 years? Express your answer in dollars/kWh.

